CONSIDERATIONS ON THE FREQUENCY DEPENDENCE OF WAVEGUIDE MODES IN PREMAGNETIZED FERRITES NEAR RESONANCE

Dietmar Köther and Adalbert Beyer, Senior Member, IEEE

Department of Electrical Engineering and SFB 254
Duisburg University, Bismarckstraße 81, D-4100 Duisburg 1, FRG

Abstract

Difficult problems occur in the design of nonreciprocal elements at millimeter wavelengths, if the dielectric and magnetic losses of ferrites are neglected. Especially near the gyromagnetic resonance the losses become large and the anisotropic behaviour of the ferrite results significant changes in the behaviour of the waveguide modes. This paper describes the solution for these waveguide modes in premagnetized ferrites and it takes into account the losses. The knowledge and better understanding of the field distribution can be used for the improvement of design procedures of high quality nonreciprocal elements. Several examples are presented and discussed to illustrate these facilities.

1. Introduction

Premagnetized ferrites are used for many applications in microwave- and millimeterwave techniques. Beside their use as tuning elements in oscillator and filter structures they are needed to realize nonreciprocal elements (isolators, circulators, phase shifters) because of the gyromagnetical properties.

In the present work electomagnetic waves in ferrite filled rectangular waveguides are investigated. Therefore the material properites of the ferrite are taken into consideration as functions of the frequency and the premagnetizing field strength. Near the gyromagnetic resonance the losses become important - which is often be neglected /1, 8/ - and the anisotropic behaviour of the material influences the electromagnetic fields significantly.

The premagnetization of the ferrite is assumed to be transversally to the direction of wave propagation. The waves calculated under these conditions will increase the understanding of tuning and nonreciprocal elements and therefore their quality can be improved.

2. Theory

In the following the ferrite material is assumed to be premagnetized in the y-direction of a cartesian coordinate system; so the permeabilty of the one axial anisotropic material can be described by the Polder-tensor /6/:

$$\overleftrightarrow{\mu} = \mu_0 \begin{bmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_{yy} & 0 \\ \mu_{zx} & 0 & \mu_{zz} \end{bmatrix} \quad , \qquad (1)$$

where μ_0 is the permeability of the vacuum.

Because of the magnetic and dielectric losses of the material the tensor elements $\mu_{xx} = \mu_{zz} = \mu_1$ and $\mu_{xz} = -\mu_{zx} = j\,\mu_2$ and the permittivity ε_r have to be inspected as complex values, that means it is valid:

$$\mu_1 = \mu_1' - j\,\mu_1'' \quad , \qquad (2)$$

$$\mu_2 = \mu_2' - j\,\mu_2'' \quad , \qquad (3)$$

$$\varepsilon_r = \varepsilon_r' - j\,\varepsilon_r'' \quad . \qquad (4)$$

Only the element $\mu_{yy}=1$ of the tensor (1) is a real value due to saturation by the high premagnetization field strength.

If the ferrite material is magnetically saturated and if the inner premagnetizing field is essentially higher than the maximum value of the magnetic AC field strength so the elements of the tensor (1) can be calculated according to Landau and Lifshitz /7/ resp. Gilbert /6/. Is the ferrite not magnetically saturated, further on the Polder-tensor can be used but its elements have to be determined by measurements or other modells.

Fig. 1 shows at the example of the ferrite material TT 2-111 (manufacturer: Trans-Tech, Inc., U.S.A.) the dependence on premagnetizing field strength of the tensor elements for a constant frequency of f = 33 GHz.

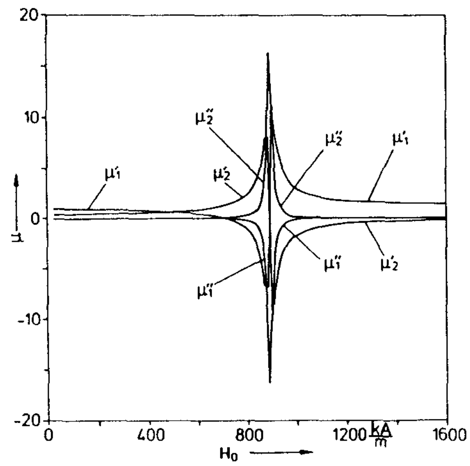

Fig. 1: The dependence of the elements of the Polder-tensor on the premagnetizing field strength H_0.

Above saturation ($H_0 > [0.1 - 0.15]\,M_s$) these functions can be calculated according the modells mentioned before /6, 7/. The phenomenological damping $\alpha_{Fe} = 0.011$ has been determined by the methode described in /2/. Below saturation of the ferrite ($H_0 < 0.05\ M_s$) an approximation has to be used /2/.

In the following, the character of waveguide modes in the presence of an anisotropic ferrite medium will be described with the help of the hybrid-mode field theory.

The rectangular waveguide to be analyzed here is represented by Fig. 2, where the cross-section is completely filled with an anisotropic medium.

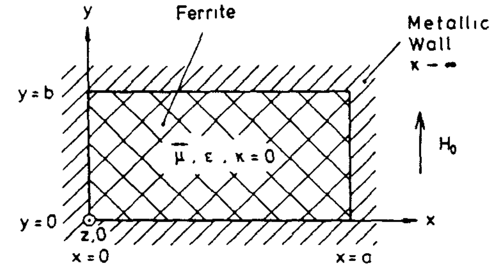

Fig. 2: Cross-section of a rectangular waveguide.

The z-dependence of all the electromagnetic waves can be described employing a linear combination of $\exp(+\gamma z)$ and $\exp(-\gamma z)$, where γ is the propagation factor, while the time dependence of the waves $\exp(j\,\omega\,t)$ will be omitted in the following considerations.

It is assumed that the non-conduction anisotropic medium ($x = 0$) is characterized by the tensor (1) of permeability $\overleftrightarrow{\mu}$ and the permittivity ε. For such media a rigorous field theory was described by Gardiol /5/ to find the x- and y-dependences of waves propagating in the above-mentioned rectangular waveguide (Fig. 2). With help of this method the components of the electromagnetic field are expanded in the following terms:

$$E_{xn}(x,y) = \tilde{E}_{xn}(x)\,\sin(k_{yn}\,y) \quad , \quad (5)$$

$$E_{yn}(x,y) = \tilde{E}_{yn}(x)\,\cos(k_{yn}\,y) \quad , \quad (5)$$

$$E_{zn}(x,y) = \tilde{E}_{zn}(x)\,\sin(k_{yn}\,y) \quad , \quad (7)$$

$$H_{xn}(x,y) = \tilde{H}_{xn}(x)\,\cos(k_{yn}\,y) \quad , \quad (8)$$

$$H_{yn}(x,y) = \tilde{H}_{yn}(x)\,\sin(k_{yn}\,y) \quad , \quad (9)$$

$$H_{zn}(x,y) = \tilde{H}_{zn}(x)\,\cos(k_{yn}\,y) \quad (10)$$

with the eigenvalue in y-direction

$$k_{yn} = \frac{n\,\pi}{b} \quad , \quad n = 0,\ 1,\ 2,\ \ldots \quad . \quad (11)$$

Under these conditions, it follows from Maxwell's equations:

$$\begin{bmatrix} 0 & -\gamma & -k_{yn} & -j\,k_0\,\mu_{xx}\,Z_{F0} & 0 & -j\,k_0\,\mu_{xz}\,Z_{F0} \\ \gamma & 0 & \frac{d}{dx} & 0 & -j\,k_0\,\mu_{yy}\,Z_{F0} & 0 \\ k_{yn} & -\frac{d}{dx} & 0 & -j\,k_0\,\mu_{zx}\,Z_{F0} & 0 & -j\,k_0\,\mu_{zz}\,Z_{F0} \\ k_0 & 0 & 0 & 0 & j\,\gamma Z_{F0} & -j\,k_{yn}\,Z_{F0} \\ 0 & k_0 & 0 & -j\,\gamma Z_{F0} & 0 & -j\,Z_{F0}\frac{d}{dx} \\ 0 & 0 & k_0 & j\,k_{yn}\,Z_{F0} & j\,Z_{F0}\frac{d}{dx} & 0 \end{bmatrix} \begin{bmatrix} \tilde{E}_{xn}(x) \\ \tilde{E}_{yn}(x) \\ \tilde{E}_{zn}(x) \\ \tilde{H}_{xn}(x) \\ \tilde{H}_{yn}(x) \\ \tilde{H}_{zn}(x) \end{bmatrix} = \vec{0} \quad (12)$$

with the wavenumber $k_0 = \omega\sqrt{\varepsilon_0\,\mu_0}$ (13)

and with the characteristic impedance of vacuum

$$Z_{F0} = \sqrt{\mu_0/\varepsilon_0} \quad . \quad (14)$$

For the simplification of eq. (12) it is convenient to define the following auxilary values

$$\mu_{eff} = \frac{\mu_{xx}\,\mu_{zz} - \mu_{xz}\,\mu_{zx}}{\mu_{xx}} = \frac{\mu_1^2 - \mu_2^2}{\mu_1} \quad , \quad (15)$$

$$v_{yx}^2 = \varepsilon_{yy}\,\mu_{xx}\,k_0^2 + \gamma^2 = \varepsilon_r\,\mu_1\,k_0^2 + \gamma^2 \quad (16)$$

and

$$v_{xy}^2 = \varepsilon_{xx}\,\mu_{yy}\,k_0^2 + \gamma^2 = \varepsilon_r\,k_0^2 + \gamma^2 \quad . \quad (17)$$

After elimination of the quantities $\tilde{E}_{xn}(x)$ and $\tilde{H}_{xn}(x)$, the eq. (12) becomes:

$$\begin{bmatrix} \varepsilon_r\,\gamma\mu_{zx}\,k_0 & \varepsilon_r\,\mu_{zx}\,k_{yn}\,k_0 & \gamma k_{yn}\,\mu_{xx} & \mu_{xx}(\varepsilon_r\,\mu_{eff}k_0^2 - k_{yn}^2) \\ 0 & 0 & -\mu_{xx}\,v_{xy}^2 & \gamma\,k_{yn}\,\mu_{xx} \\ -\varepsilon_r\,\gamma k_{yn} & \varepsilon_r(\varepsilon_r\,\mu_{xx}\,k_0^2 - k_{yn}^2) & 0 & \varepsilon_r\,\mu_{xy}\,k_{yn}\,k_0 \\ -\varepsilon_r\,v_{yx}^2 & -\varepsilon_r\,\gamma k_{yn} & 0 & \gamma\mu_{xz}\,k_0 \end{bmatrix} \cdot \frac{1}{\varepsilon_r\,\mu_{xx}\,k_0} \begin{bmatrix} \tilde{E}_{yn}(x) \\ \tilde{E}_{zn}(x) \\ -jZ_{F0}\tilde{H}_{yn}(x) \\ -jZ_{F0}\tilde{H}_{zn}(x) \end{bmatrix} = \frac{d}{dx} \begin{bmatrix} \tilde{E}_{yn}(x) \\ \tilde{E}_{zn}(x) \\ -jZ_{F0}\tilde{H}_{yn}(x) \\ -jZ_{F0}\tilde{H}_{zn}(x) \end{bmatrix} \ . \quad (18)$$

It is a system of homogeneous differential equations

$$\overleftrightarrow{A}_n\,\vec{F}_n(x) = \frac{d}{dx}\,\vec{F}_n(x) \quad (19)$$

with $\vec{F}_n(x)$ the amplitude vector and $\overleftrightarrow{A}_n$ the system matrix. From the solutions of this differential equation the unknown amplitude functions $\tilde{E}_{xn}(x)$, ..., $\tilde{H}_{zn}(x)$ can be determined /3,4,5/.

Regarding to the boundary conditions at $x = 0$, for a particular solution of the differential equation (18) yields:

$$\vec{F}_n(x) = \vec{F}_n(x{=}0)\,\exp(\overleftrightarrow{A}_n\,x) \quad . \quad (20)$$

From this the electromagnetic fields can be calculated by use of the method descibed in /4/.

3. Numerical Results

To classify the waves a name convention is introduced which takes into account their field distribution. For waveguide modes with a field behaviour similar to that of homogeneous and isotropic rectangular waveguides a classification is used corresponding to the well-known division into TM_{mn}- and TE_{mn}- modes for this rectangular waveguides.

Fig. 3 shows the dispersion behaviour ($\gamma = \alpha + j\,\beta$) of the fundamental mode (TE_{10}) in a rectangular waveguide completely filled with a lossy ferrite material.

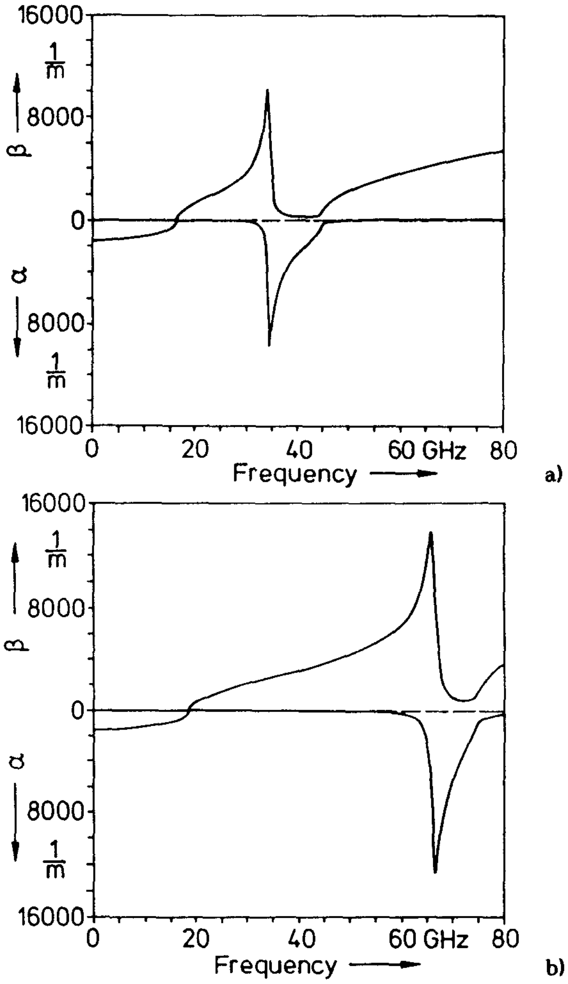

Fig. 3: Dispersion behaviour of the fundamental mode (TE_{10}) in a rectangular waveguide completely filled with the lossy ferrite material TT 2-111 (Dimensions: a = 2 mm, b = 1 mm).
a) H_0 = 750 kA/m, b) H_0 = 1600 kA/m.

The diagrams of Fig. 3 have been calculated under consideration of the results of Fig. 1. This kind of wave suffers strong damping near the gyromagnetic resonance, whereby the characteristic values α and β stay finite because of the losses. But the maxima does not occur at the same frequencies because the damping values α are enforced by the magnetic losses while the phase constant β becomes large when the tensor element μ_1 nearly vanishes. With increase of the premagnetizing field strength the gyromagnetic resonance frequency shifts, too and the maxima of the functions α and β become more significant. During this the cut-off frequency increases from 15.2 GHz to 19 GHz. For small values of the premagnetizing feld strength only little values of β occur near the gyromagnetic resonance. With increase of H_0 the value of β increases, too.

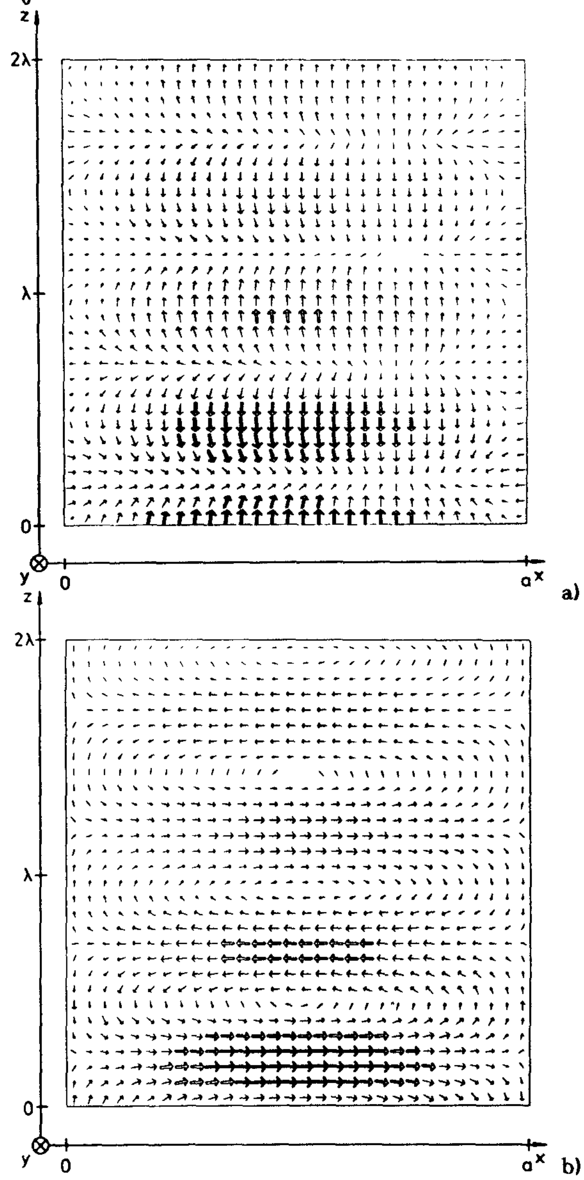

Fig. 4: Field distribution of the fundamental mode (TE_{10}) for H_0 = 750 kA/m. Distribution of: a) the magnetic field strength, b) the magnetic flux density.

Fig. 4 shows the longitudinal distribution of the magnetic field strength and the magnetic flux density for the fundamental wave at 33 GHz (near gyromagnetic resonance). While the magnetic flux density remains source free (Fig. 4a) the magnetic field strength has sources. These sources are placed on planes travelling into the z-direction. The field lines of the magnetic field strength are almost perpendicular to the magnetic flux density because $\mu_1 \approx 0$.

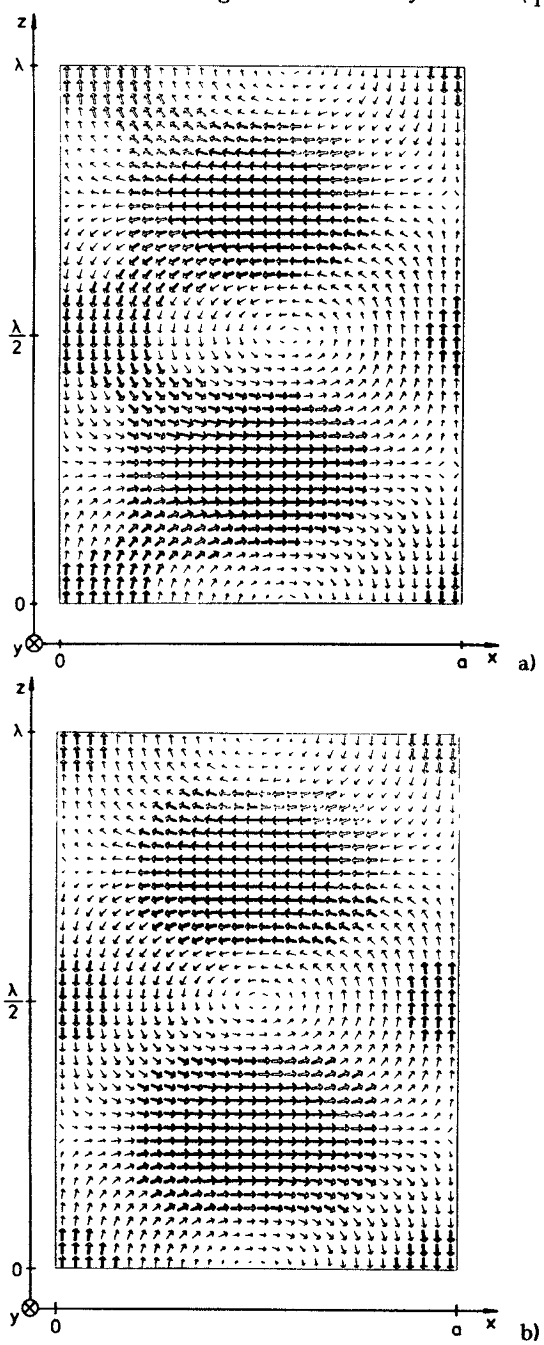

Fig. 5: Field distribution of the fundamental mode (TE_{10}) for H_0 = 1600 kA/m. Distribution of: a) the magnetic field strength, b) the magnetic flux density.

Fig. 5 shows the according field distributions for a higher premagnetizing field (1600 kA/m). It can be seen that even for such large fields the ferrite does not become isotropic. The distribution of the magnetic flux density is very similar to that of a lossless rectangular waveguide homogeneous filled with an isotropic medium (Fig. 5b). The magnetic field strength is disturbed and has sources at the right side near the metallic wall (Fig. 5a). Meanwhile the tensor element μ_2 vanishes.

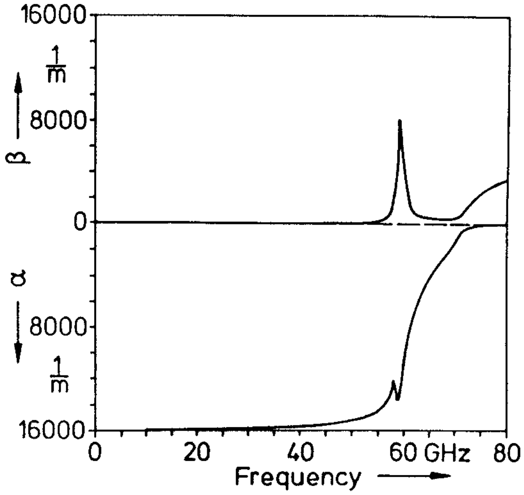

Fig. 6: Dispersion behaviour of the first higher order mode in a rectangular waveguide filled with lossy ferrite material (H_0 = 1600 kA/m).

The first higher order mode is strongly damped below the gyromagnetic resonance frequency and is therefore not a propagation wave. Near the gyromagnetic resonance a local maximum of α is corresponding with a maximum of β. Both maxima are enforced by the magnetic losses even below cut-off frequency.

4. Conclusion

This paper has shown that the field distributions inside an anisotropic medium-filled rectangular waveguide can be calculated on the basis of a rigorous field theory. Furthermore, it has been pointed out that the characteristics of waveguide modes can also be described. The above field theory can be used for the design of high quality isolators and phase shifters.

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